

## COMMON ION EFFECT – WORKED PROBLEMS

The solubility of insoluble substances can be decreased by the presence of a common ion. AgCl will be our example.

Present in silver chloride are silver ions ( $\text{Ag}^+$ ) and chloride ions ( $\text{Cl}^-$ ). Silver nitrate (which is soluble) has silver ion in common with silver chloride. Sodium chloride (also soluble) has chloride ion in common with silver chloride.

In fact, mixing sufficiently concentrated solutions of  $\text{AgNO}_3$  and  $\text{NaCl}$  will produce a precipitate of AgCl. In order to be sufficiently concentrated, the product of the  $[\text{Ag}^+]$  and the  $[\text{Cl}^-]$  must exceed the  $K_{\text{sp}}$  of  $1.77 \times 10^{-10}$ .

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**Example #1:** AgCl will be dissolved into a solution with is ALREADY 0.0100 M in chloride ion. What is the solubility of AgCl?

By the way, the source of the chloride is unimportant (at this level). Let us assume the chloride came from some dissolved sodium chloride, sufficient to make the solution 0.0100 M. So, on to the solution . . .

The dissociation equation for AgCl is:



The  $K_{\text{sp}}$  expression is:

$$K_{\text{sp}} = [\text{Ag}^+] [\text{Cl}^-]$$

This is the equation we must solve. First we put in the  $K_{\text{sp}}$  value:

$$1.77 \times 10^{-10} = [\text{Ag}^+] [\text{Cl}^-]$$

Now, we have to reason out the values of the two guys on the right. The problem specifies that  $[\text{Cl}^-]$  is already 0.0100. I get another 'x' amount from the dissolving AgCl. Of course,  $[\text{Ag}^+]$  is 'x.'

Substituting, we get:

$$1.77 \times 10^{-10} = (x) (0.0100 + x)$$

This will wind up to be a quadratic equation which is solvable via the quadratic formula. However, there is a chemical way to solve this problem. We reason that 'x' is a small number, such that '0.0100 + x' is almost exactly equal to 0.0100. If we were to use

0.0100 rather than '0.0100 + x,' we would get essentially the same answer and do so much faster. So the problem becomes:

$$1.77 \times 10^{-10} = (x) (0.0100)$$

and

$$x = 1.77 \times 10^{-8} \text{ M}$$

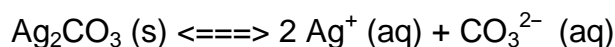
There is another reason why neglecting the 'x' in '0.0100 + x' is OK. It turns out that measuring  $K_{sp}$  values are fairly difficult to do and, hence, have a fair amount of error already built into the value. So the very slight difference between 'x' and '0.0100 + x' really has no bearing on the accuracy of the final answer. Why not? Because the  $K_{sp}$  already has significant error in it to begin with. Our "adding" a bit more error is insignificant compared to the error already there.

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**Example #2:** The  $K_{sp}$  for silver carbonate is  $8.4 \times 10^{-12}$ . The concentration of carbonate ions in a saturated solution is  $1.28 \times 10^{-4}$  M. What is the concentration [of silver](#) ions?

**Solution:**

Dissociation equation:



$K_{sp}$  expression:

$$K_{sp} = [\text{Ag}^+]^2 [\text{CO}_3^{2-}]$$

Let us substitute into the  $K_{sp}$  expression:

$$8.4 \times 10^{-12} = (x)^2 (1.28 \times 10^{-4})$$

Note: I could have used  $(1.28 \times 10^{-4} + 0.5x)$  for the carbonate. See above discussion for why the  $0.5x$  can be dropped.

Divide both sides by  $1.28 \times 10^{-4}$  and then take the square root:

$$[\text{Ag}^+] = x = 2.56 \times 10^{-4} \text{ M}$$

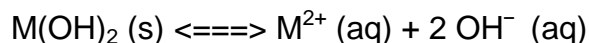
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**Example #3:** The molar solubility of a generic substance,  $\text{M}(\text{OH})_2$  in 0.10 M KOH solution is  $1.0 \times 10^{-5}$  mol/L. What is the  $K_{sp}$  for  $\text{M}(\text{OH})_2$ ?

**Solution:**

In this case, we are being asked for the  $K_{sp}$ , so that is where our 'x' will be. That means the right-hand side of the  $K_{sp}$  expression (where the concentrations are) cannot have an unknown.

Dissociation equation:



$K_{sp}$  expression:

$$K_{sp} = [M^{2+}] [OH^-]^2$$

Let us substitute into the  $K_{sp}$  expression:

$$x = (1.0 \times 10^{-5}) (0.10)^2$$

The  $1.0 \times 10^{-5}$  comes from the molar solubility information, coupled with the fact that for every one  $M(OH)_2$ , one  $M^{2+}$  is produced.

Also, we could have used  $(0.10 + 2.0 \times 10^{-5})$  M for the  $[OH^-]$ . However, the  $2.0 \times 10^{-5}$  M, being much smaller than 0.10, is generally ignored.

The answer:

$$K_{sp} = 1.0 \times 10^{-7}$$

**Example #4:** What is the solubility of AgI in a 0.274-molar solution of NaI. ( $K_{sp}$  of AgI =  $8.52 \times 10^{-17}$ )

**Solution:**

Dissociation equation:



$K_{sp}$  expression:

$$K_{sp} = [Ag^+] [I^-]$$

Let us substitute into the  $K_{sp}$  expression:

$$8.52 \times 10^{-17} = (x) (0.274)$$

The answer:

$$[\text{Ag}^+] = 3.11 \times 10^{-16} \text{ M}$$

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**Example #5:** What is the solubility of  $\text{Ca}(\text{OH})_2$  in 0.0860 M  $\text{Ba}(\text{OH})_2$ ?

**Solution:**

$\text{Ba}(\text{OH})_2$  is a strong base so  $[\text{OH}^-] = 2 \text{ times } 0.0860 = 0.172 \text{ M}$

Dissociation equation:



$K_{\text{sp}}$  expression:

$$K_{\text{sp}} = [\text{Ca}^{2+}] [\text{OH}^-]^2$$

The  $K_{\text{sp}}$  for  $\text{Ca}(\text{OH})_2$  is known to be  $4.68 \times 10^{-6}$ . We set  $[\text{Ca}^{2+}] = x$  and  $[\text{OH}^-] = (0.172 + 2x)$ . Substituting into the  $K_{\text{sp}}$  expression:

$$4.68 \times 10^{-6} = (x) (0.172 + 2x)^2$$

Ignoring the "2x," we find  $x = 1.58 \times 10^{-4} \text{ M}$

Comment: There are several different values floating about the Internet for the  $K_{\text{sp}}$  of  $\text{Ca}(\text{OH})_2$ . I got mine from the CRC Handbook, 73rd Edition, pg. 8-43.

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**Example #6:** The solubility product of  $\text{Mg}(\text{OH})_2$  is  $1.2 \times 10^{-11}$ . What minimum  $\text{OH}^-$  concentration must be attained (for example, by adding  $\text{NaOH}$ ) to decrease the  $\text{Mg}^{2+}$  concentration in a solution of  $\text{Mg}(\text{NO}_3)_2$  to less than  $1.1 \times 10^{-10} \text{ M}$ ?

**Solution:**

$K_{\text{sp}}$  expression:

$$K_{\text{sp}} = [\text{Mg}^{2+}] [\text{OH}^-]^2$$

We set  $[\text{Mg}^{2+}] = 1.1 \times 10^{-10}$  and  $[\text{OH}^-] = x$ . Substituting into the  $K_{\text{sp}}$  expression:

$$1.2 \times 10^{-11} = (1.1 \times 10^{-10}) (x)^2 \quad x = 0.33 \text{ M}$$

Any sodium hydroxide solution greater than 0.33 M will reduce the  $[\text{Mg}^{2+}]$  to less than  $1.1 \times 10^{-10}$  M.

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**Example #7:** Calculate the pH at which zinc hydroxide just starts to precipitate from a 0.00857 M solution of zinc nitrate.  $K_{\text{sp}}$  for zinc hydroxide =  $3.0 \times 10^{-17}$

**Solution:**

1)  $K_{\text{sp}}$  expression:

$$K_{\text{sp}} = [\text{Zn}^{2+}] [\text{OH}^-]^2$$

2) Substitute and solve for  $[\text{OH}^-]$ :

$$3.0 \times 10^{-17} = (0.00857) (x)^2$$

$$x = 5.91657 \times 10^{-8} \text{ M (I kept a few guard digits.)}$$

3) Compute the pH:

$$\text{pOH} = 7.228$$

$$\text{pH} = 6.772$$

Note how zinc hydroxide would precipitate even when the solution is slightly acidic.

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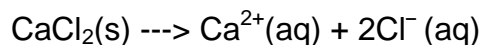
**Example #8:** What is the solubility, in moles per liter, of AgCl ( $K_{\text{sp}} = 1.77 \times 10^{-10}$ ) in 0.0300 M  $\text{CaCl}_2$  solution?

**Solution:**

1) Concentration of chloride ion from calcium chloride:

$$0.0300 \text{ M} \times 2 = 0.0600 \text{ M}$$

from here:



2) Calculate solubility of  $\text{Ag}^+$ :

$$K_{\text{sp}} = [\text{Ag}^+] [\text{Cl}^-]$$

$$1.77 \times 10^{-10} = (x) (0.0600)$$

$$x = 2.95 \times 10^{-9} \text{ M}$$

Since there is a 1:1 ratio between the moles of aqueous silver ion and the moles of silver chloride that dissolved,  $2.95 \times 10^{-9} \text{ M}$  is the molar solubility of AgCl in 0.0300 M CaCl<sub>2</sub> solution.

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**Example #9:** Calculate the number of moles of Ag<sub>2</sub>CrO<sub>4</sub> that will dissolve in 1.00 L of 0.010 M K<sub>2</sub>CrO<sub>4</sub> solution.  $K_{sp}$  for Ag<sub>2</sub>CrO<sub>4</sub> =  $9.0 \times 10^{-12}$ .

**Solution:**

1) Concentration of dichromate ion from potassium chromate:

$$0.010 \text{ M}$$

2) Calculate solubility of Ag<sup>+</sup>:

$$K_{sp} = [\text{Ag}^+]^2 [\text{CrO}_4^{2-}]$$

$$9.0 \times 10^{-12} = (x)^2 (0.010)$$

$$x = 3.0 \times 10^{-5} \text{ M}$$

Since there is a 2:1 ratio between the moles of aqueous silver ion and the moles of silver chromate that dissolved,  $1.5 \times 10^{-5} \text{ M}$  is the molar solubility of Ag<sub>2</sub>CrO<sub>4</sub> in 0.010 M K<sub>2</sub>CrO<sub>4</sub> solution.

Since we were asked for the moles of silver chromate that would dissolve in 1.00 L, the final answer is:

$$1.5 \times 10^{-5} \text{ mol}$$

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**Example #10:** What is the maximum concentration of Mg<sup>2+</sup> ion that can remain dissolved in a solution that contains 0.7147 M NH<sub>3</sub> and 0.2073 M NH<sub>4</sub>Cl? ( $K_{sp}$  for Mg(OH)<sub>2</sub> is  $1.2 \times 10^{-11}$ ;  $K_b$  for NH<sub>3</sub> is  $1.77 \times 10^{-5}$ )

**Solution:**

1) Use the acid base data supplied to calculate [OH<sup>-</sup>]:

$$K_b = \frac{[\text{NH}_4^+][\text{OH}^-]}{[\text{NH}_3]}$$

$$1.77 \times 10^{-5} = [(0.2073)(x)] / 0.7147$$

$$x = 6.10 \times 10^{-5} \text{ M}$$

2) Use the  $K_{sp}$  expression to calculate the  $[\text{Mg}^{2+}]$ :

$$K_{sp} = [\text{Mg}^{2+}] [\text{OH}^-]^2$$

$$1.2 \times 10^{-11} = (x) (6.10 \times 10^{-5})^2$$

$$x = 3.2 \times 10^{-3} \text{ M}$$

**Example #11:**  $1.1 \times 10^{-4} \text{ g}$  of  $\text{Cr}(\text{OH})_3$  is added to 120 L of water at 25 °C. Will it all dissolve? ( $K_{sp} = 6.7 \times 10^{-31}$ )

**Solution:**

1) Solve  $K_{sp}$  expression for the molar solubility:

$$K_{sp} = [\text{Cr}^{3+}] [\text{OH}^-]^3$$

$$6.7 \times 10^{-31} = (x) (3x)^3$$

$$x = 1.255 \times 10^{-8} \text{ M}$$

2) Convert to grams per liter:

$$1.255 \times 10^{-8} \text{ mol/L times } 103.0 \text{ g/mol} = 1.29 \times 10^{-6} \text{ g/L}$$

3) Check the problem's data:

$$1.1 \times 10^{-4} \text{ g} / 120 \text{ L} = 9.17 \times 10^{-7} \text{ g/L}$$

All the  $\text{Cr}(\text{OH})_3$  dissolves.

**Example #11 - Part 2:**  $4.0 \times 10^{-4} \text{ g}$  of NaOH is added. Will a precipitate form?

**Solution:**

1) Convert g/L to mol/L:

$$\text{Cr}(\text{OH})_3 \\ 9.17 \times 10^{-7} \text{ g/L divided by } 103 \text{ g/mol} = 8.90 \times 10^{-9} \text{ M}$$

NaOH

$$4.0 \times 10^{-4} \text{ g} / 120 \text{ L} = 3.33 \times 10^{-6} \text{ g/L}$$

$$3.33 \times 10^{-6} \text{ g/L divided by } 40.0 \text{ g/mol} = 8.33 \times 10^{-8} \text{ M}$$

2) Calculate a  $Q_{sp}$ :

$$x = (8.90 \times 10^{-9}) (8.33 \times 10^{-8})^3$$

$$x = 5.15 \times 10^{-30}$$

$\text{Cr}(\text{OH})_3$  precipitates.

**Example #11 - Part 3:** Calculate the molar solubility of  $\text{Cr}(\text{OH})_3$  in a solution buffered at  $\text{pH} = 11.00$

**Solution:**

1) Determine the  $[\text{OH}^-]$ :

$$\text{pOH} = 14.00 - 11.00 = 3.00$$

$$[\text{OH}^-] = 10^{-\text{pOH}} = 10^{-3.00} = 1.0 \times 10^{-3} \text{ M}$$

2) Determine the molar solubility:

$$6.7 \times 10^{-31} = (x) (1.0 \times 10^{-3})^3$$

$$x = 6.7 \times 10^{-22} \text{ M}$$